Appendix J.

Differential polarization calculations using switched transmit phase

1. INTRODUCTION

This document discusses algorithms that can be used for calculating differential polarization. The algorithms seen in the literature are for equally spaced pulses, and so must be modified slightly for use with pairs of pulses that are unequally spaced. This document also discusses some of the trade-offs involved between different polarization sequences and single and dual-receiver systems. Some of the functions that are used in the signal processing code are presented.

2. ALGORITHMS

Measurement of rainfall by the differential phase method is based on the fact that horizontally and vertically polarized waves propagate through rainfall at different rates, which causes a phase shift between them. By measuring the phase shift over range, and then looking at how fast the phase is shifting vs. range, the rainfall rate can be inferred. A dual receiver/transmitter system that can transmit and receive both polarizations simultaneously can make this measurement by comparing the phases between the horizontal and vertical channels. With only one transmitter, some time elapses between the horizontal and vertical sampling, and thus the phase is changed by the Doppler shift. This must be accounted for in calculating differential phase. The following algorithms account for this effect in their derivation.

2.1 Calculation of differential phase and derived products

2.1.1 Calculation of differential phase from a single receiver

The following is from Sachidananda and Zrnic (1989), and assumes a single receiver.

The differential phase algorithm for an equally spaced transmit sequence of HVHV... and a corresponding receive sequence of hvhv... requires forming sums of the form

$$R_a(T_s) = \frac{1}{M}\sum_{i=0}^{M-1} H_{2i}^{*} V_{2i+1} \ , \qquad R_b(T_s) = \frac{1}{M}\sum_{i=0}^{M-1} V_{2i+1}^{*} H_{2i+2} \ ;$$

where H and V, which are complex, represent the echo from an H- or V-transmitted wave, M is half the number of pulses in the beam, and i is the pulse index. Note that H and V in the equations are redundant, since whether i is odd or even specifies whether the signal is from an H or V pulse; however, their use clarifies what is being done.

Differential phase, as defined by Φ_{HH} - Φ_{VV}, is estimated from

$$\Phi_{DP} = \frac{1}{2} \mathit{arg}(R_a R_b^*) \ .$$

Φ_{DP} is a generally increasing function with range when waves propagate through most types of hydrometeors. The preceding equation only returns arguments from $\pm\pi$, so it will be necessary to monitor when the phase folds and add 2π to it to provide a continuous function.

2.1.2 Calculation of K_{DP}

K_{DP} is the range derivative of Φ_{DP} , but Φ_{DP} data is usually too noisy to take the derivative directly. Instead, filtered derivatives must be used.

2.1.3 Calculation of Rainfall Rate

From Jameson (1991), Table 2, there is the following relationship:

$$R = C\Phi^p \ ,$$

where R is rainfall rate in mm/hr, and Φ is phase shift in deg/km, which is equivalent to our K_{DP}. Using the numbers in his table for 9 GHz:

$$R = 13.03 K_{DP}^{0.9403} \ .$$

Using this (or similar relationships), plots of rainfall vs. range can be produced.

2.2 Calculation of ρ_{hv}

The following is from Zahrai and Zrnic (1993), pp. 653 - 655.

ρ_{hv} represents the correlation between signals in Hh and Vv. Because simultaneous Hh and Vv samples are not available, some assumptions were made, including that of a Gaussian spectrum shape, to derive the following equations.

First, compute

$$S_h = \frac{1}{M} \sum_{i=1}^{M} \left| H_{2i} \right|^2 , \qquad S_v = \frac{1}{M} \sum_{i=1}^{M} \left| V_{2i+1} \right|^2 .$$

Then compute the following running-sum

$$\rho(2T_s) = \frac{\left| \sum_{i=1}^{M} \left(H_{2i}^* H_{2i+2} + V_{2i+1}^* V_{2i+3} \right) \right|}{M(S_h + S_v)} .$$

Then compute

$$\left|\rho_{hv}(T_s)\right| = \frac{|R_a| + |R_b|}{2\sqrt{S_h S_v}}.$$

Finally,

$$\left|\rho_{hv}(0)\right| = \frac{\left|\rho_{hv}(T_s)\right|}{\left[\rho(2T_s)\right]^{0.25}}.$$

The paper states that this estimator is biased by white noise, and that the bias can be removed if the signal-to-noise ratio is known; however, the formula to do this is not given.

2.3 Calculation of Velocity and other Pulse Pair Products

Velocities can be calculated by running HV data through the standard pulse pair algorithm with overlapping pairs, but there are problems: the velocity is only unambiguous when $\Phi_{DP} < 90°$, and the estimator becomes increasingly noisy as it approach this value. There are ways to correct for these effects, but they add computational complexity. For this application, a better way is to process the data as pairs (*e.g.,* HH VV HH VV) with the same received polarization within the pair such that our standard estimators for velocity, width, power and correlation can be used. This is discussed in the next section.

2.4 Algorithm Modifications when using Pulse Pairs

2.4.1 Polarization Sequences

If we restrict ourselves to having the same transmit and receive polarization within a pair, some simplifications can be made in the programming. We will always number our pulses starting with zero, so the last pulse is the N - 1 pulse. Pulse sequences will repeat after m pulses, so N/m must be an integer. Because it is desired to obtain all possible co- and cross-polarization products, four pairs of pulses will be needed. In order to have the products in the equations for R_a and R_b come from equally spaced samples, the co-polar pairs must be equally spaced. If we choose to first transmit horizontally, then the sequence must look like this:

H	H			V	V			H	H			V	V		
h	h			v	v			h	h			v	v		
0	1	2	3	4	5	6	7	8	9	1	1	1	1	1	1
										0	1	2	3	4	5

The missing cross-polar pairs can be filled in two different ways, and we have chosen the following:

H	H	H	H	V	V	V	V	H	H	H	H	V	V	V	V
h	h	v	v	v	v	h	h	h	h	v	v	v	v	h	h
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15

This illustrates a sequence where N = 16 and m = 8, and is referred to as the primary polarization sequence.

2.4.2 Differential Phase Algorithms

The preceding algorithms, which are given for an equally spaced pulse train of HVHV... and hvhv..., must be modified for use with pulse pair. Use as an example the following sums:

$$R_a(T_s) = \frac{1}{M}\sum_{i=0}^{M-1} H^*_{2i} V_{2i+1}, \qquad R_b(T_s) = \frac{1}{M}\sum_{i=0}^{M-1} V^*_{2i+1} H_{2i+2} .$$

Using the primary polarization sequence, we want to form our R_a sum from the factors (0,4) and (1,5); and our R_b from the factors (4,8) and (5,9). Both pulses in the pair are used to reduce the uncertainty of the estimate. Since similar sums are used in several places, it is desirable to define a general function for these covariance functions that will operate on an arbitrary polarization sequence. If we let E_i represent the i^{th} complex sample in the sequence, then we can write

$$R_p(s,n,m,N) = \frac{m}{2(N-m)}\sum_{i=0}^{\frac{N}{m}-2} (E^*_{s+mi} E_{s+n+mi} + E^*_{s+1+mi} E_{s+1+n+mi}),$$

where s is the starting index, n is the lag, m is the length of the sequence cycle, and N is the total length of the sequence (number of triggers). The "p" in R_p indicates the data were processed as pairs of pulse pairs. Then

$R_a = R_p(0, 4, 8, 16)$

in our example of a 16-pulse sequence, and

$R_b = R_p(4, 4, 8, 16)$.

Also, the sum

$$\sum_{i=1}^{M}\left(H^*_{2i} H_{2i+2} + V^*_{2i+1} V_{2i+3}\right)$$

can be represented as

$R_p(0, 8, 8, 16) + R_p(4, 8, 8, 16)$.

2.4.3 The Pulse Pair Algorithm

We can also use a similar scheme for the standard pulse-pair algorithm. The running sums for the algorithm can be written as

$$R(T_s)=\frac{2}{N}\sum_{i=0}^{\frac{N}{2}-1}E_{2i}E^*_{2i+1} \qquad R(0)=\frac{1}{N}\sum_{i=0}^{\frac{N}{2}-1}\left(\left|E_{2i}\right|^2+\left|E_{2i+1}\right|^2\right)$$

for a polarization sequence such as HH HH HH HH ....
For a generalized polarization sequence, we can define functions

$$R_s(s,m,N)=\frac{m}{N}\sum_{i=0}^{\frac{N}{m}-1}E_{s+im}E^*_{s+1+im} \text{ and } R_0(s,m,N)=\frac{m}{2N}\sum_{i=0}^{\frac{N}{m}-1}\left(\left|E_{s+im}\right|^2+\left|E_{s+1+im}\right|^2\right).$$

These functions can be used to compute any of the running sums from a specified polarization pair in the sequence.

3. HARDWARE TRADEOFFS

The following discussion concerns what can be done within a single beam. Note that H and V refer to transmitted polarization, and h and v refer to received polarization.

3.1 Single Receiver

The non-pulse pair transmit sequence HVHV... gives the shortest dwell time, but it has some problems. In applying a modified pulse-pair estimator to this sequence, the unambiguous velocity is cut in half. There are methods to deal with this, but they add computational complexity. Since the receiver polarization is hvhv..., less than half the data products possible are produced, and the velocity is a mix of horizontally and vertically polarized co-polar data.

The next possibility is to use pulse pair and transmit a sequence of HH VV HH VV and a matching receive sequence of hh vv hh vv. Now the pulse pair algorithm on the HH and VV pairs can be used separately to obtain co-polar velocity for both polarizations with the expected unambiguous velocity; however, only half of the available data products are obtained as there are no cross-polarized products. The dwell time is twice as long as the previous case for pulse-pair products (although co- and cross-polarized data are being mixed together in the first case). The dwell time for differential phase products is the same as in the first case, but the time between the differential phase samples is twice as long.

Another possibility is to use pulse pair and transmit HH HH VV VV and receive hh vv vv hh. The differential phase products are formed by combining data from the first and third pairs. Now all four data products are obtained: Hh, Hv, Vh and Vv. The dwell time for pulse-pair products is four times as long as in the first case. The dwell time for differential phase products is twice as long as in the first and second cases. The time between samples for differential phase is four times as long as in the first case, and twice as long as in the second case, which may have some implications for the differential phase estimators.

3.2 Dual Receiver

A dual receiver has the inherent problem of matching phase and amplitude characteristics between the two receiver channels, but this has not proven difficult in practice. Looking vertically in rain can be used as a validation of calibration, since the circular cross-section of the rain drops provides a signal that should be matched between the two channels.

The advantage of the dual receiver approach is that the data are gathered twice as fast. The obvious choice for a transmit polarization sequence is HH VV HH VV while receiving hh and vv simultaneously all the time.

3.3 Dual Receiver -- Dual Transmitter

Being able to transmit and receive both polarizations simultaneously is the ultimate polarization capability, but requires two transmitter tubes or a high-power signal splitter. The CHILL S-band radar has this capability. This provides a further factor-of-two speed up in data acquisition. Unfortunately, this is not possible with our existing equipment configuration.

3.4 Summary

The following table summarizes the characteristics of different polarization sequences. For comparison purposes, everything is compared to an eight-pulse sequence, even though some sequences repeat after two or four pulses. B or b represents both polarizations simultaneously. The beam time is for an average PRP of 500 μs.

Table 1. Summary of different polarization sequences.

Polarization sequence	Polarization products	Pulse pair samples/ sequence/product	Beam time for 128 samples	Comment
HVHVHVHV hvhvhvhv	1	8	64 ms	Pulse-pair polarization products mixed together
HH VV HH VV Hh vv hh vv	2	4	128 ms	Only co-polar products
HH HH VV VV Hh vv vv hh	4	2	256 ms	co- and cross-polar products
HH VV HH VV Bb bb bb bb	4	4	128 ms	dual receiver, all products
BB BB BB BB Bb bb bb bb	4	8	64 ms	dual receiver and transmitter, all products

4. DATA PRODUCTS

4.1 Nomenclature

The covariance (running-sums) algorithms run through an input data stream and combine variables in various positions in certain ways without taking into account the polarization used to gather these variables. Thus, it is useful to define variables at this point that only depend on their position in the sequence. There are three types of covariance variables: Rs, Rp and Rz.

Rs represents variables where the conjugate product is formed between two samples in the same pair. These are used for the standard pulse-pair algorithm where the lag is always one sample time, and the sum is over the same polarizations. There are at most four of these complex variables: RsHh, RsHv, RsVv and RsVh. The generic (polarization-independent) form of these variables is Rsi, where $0 \leq i \leq m - 2$ and i represents the first location in the sequence of the variable. The angle of these variables times a constant gives the velocity.

Rp represents variables where the conjugate product is formed between samples in different pairs. In this case, it is always desirable to form a conjugate product between both sets of pulses in the two pairs in order to reduce the uncertainty of the estimate. The algorithms specified so far require the formation of four of these complex variables: RpVH, RpHV, RpHH and RpVV. The two capital letters following Rp imply that the transmitted and received polarizations are the same. If we wanted to form cross-polarized products, more letters would be required, e.g., RpHvVh. The generic form of these variables is Rpin, where i represent the first location in the sequence of the variable, and n represents the number of lags to the second variable.

Rz represents an autocovariance at zero lag, which is the square of the magnitude of the complex sample. When we are performing this operation, we always want to use both pulses in the pair. There are four possible values of this variable: RzHh, RzHv, RzVv and RzVh. The sequence-independent form of this variable is Rzi.

4.2 Summary of Naming Rules for Variables

R indicates the covariance function. If it is followed by an s, it indicates that the product was done within the pair and the following letters indicate the transmit/receive polarization (this should never change within a pair). If it is followed by a p, it indicates that the product was done from one pair to another, and both samples in the pair were used to reduce the uncertainty. If both of the following letters are capitals, it indicates the transmit polarization and implies that the received polarization was the same.

If the R is followed by a z, it indicates a zero-lag autocovariance (proportional to power). Both samples in the pair are used to form this quantity. The next two letters indicate the transmit/receive polarization.

4.3 Products for Recording

As is done for the normal pulse-pair mode, only covariance (running-sums) data is recorded, both to save tape and to allow recalculation of derived fields with different parameters. All fields are DC-corrected. The required fields are shown in the following table. Complex quantities are in **bold** type.

Table 2. Recorded Products

Generic variable name	Polarization variables	Mathematical symbol	Function used to calculate variable
Rs0	**RsHh**	$\mathbf{R(T_s)}$	**Rs**(0, 8, N)
Rs2	**RsHv**	$\mathbf{R(T_s)}$	**Rs**(2, 8, N)
Rs4	**RsVv**	$\mathbf{R(T_s)}$	**Rs**(4, 8, N)
Rs6	**RsVh**	$\mathbf{R(T_s)}$	**Rs**(6, 8, N)
Rz0	RzHh	R(0) or S_h	Rz(0, 8, N)
Rz2	RzHv	R(0)	Rz(2, 8, N)
Rz4	RzVv	R(0) or S_v	Rz(4, 8, N)
Rz6	RzVh	R(0)	Rz(6, 8, N)
Rp04	**RpHV**	$\mathbf{R_a(T_s)}$	**Rs**(0, 4, 8, N)
Rp44	**RpVH**	$\mathbf{R_b(T_s)}$	**Rs**(4, 4, 8, N)
Rp08	**RpHH**	$\sum \mathbf{H^*_{2i}\, H_{2i+2}}$	**Rs**(0, 8, 8, N)
Rp48	**RpVV**	$\sum \mathbf{V^*_{2i+1}\, V_{2i+3}}$	**Rs**(4, 8, 8, N)

4.4 Products for Display

Using the polarization sequence HH HH VV VV and hh vv vv hh, the following basic quantities from the linear channel can be calculated using the standard relations.

Table 3. Pulse-Pair Variables.

Field					**Units**
Velocity	VPHh	VPHv	VPVv	VPVh	m/s
Width	WPHh	WPHv	WPVv	WPVh	m^2/s^2
Correlation	CPHh	CPHv	CPVv	CPVh	none
Intensity	NIHh	NIHv	NIVv	NIVh	watts at receiver output
Power	NPHh	NPHv	NPVv	NPVh	dBm at antenna terminals
Reflectivity	NZHh	NZHv	NZVv	NZVh	dBZ

These quantities can be combined to compute the following:

Z_{DR} = DZHV = 10 log NIHH - 10 log NIVV

LDR_{hv} = DIVhVv = 10 log NIVh - 10 log NIVv

DDV = DVVH = (VPVV - VPHH)/sin(elev)

Of course, numerous other combinations could be computed as well, if so desired.

Using the previously described algorithms, Φ_{DP}, R and $\rho_{hv}(0)$ can be calculated. The existing in-house algorithm can be used to compute K_{DP}.

The list of 32 quantities available for display is as follows:

Table 4. Display Variables.

Description					**Units**
Velocity	VPHh	VPHv	VPVv	VPVh	m/s
Width	WPHh	WPHv	WPVv	WPVh	m^2/s^2
correlation	CPHh	CPHv	CPVv	CPVh	none
intensity (power at rcvr output)	NIHh	NIHv	NIVv	NIVh	watts
power (power at antenna terminals)	NPHh	NPHv	NPVv	NPVh	dBm
reflectivity	NZHh	NZHv	NZVv	NZVh	dBZ
differential phase, raw	PHIdp				deg
differential phase, smoothed	PHIdps				deg
differential propagation constant	Kdp				deg/km
rainfall rate	Rain				mm/hr
correlation coefficient, H and V	RHOHV				none
differential reflectivity	DZHV				dB
linear depolarization ratio	DIVhVv				dB
differential Doppler velocity	DVVH				m/s

5. SUMMARY

Based on the preceding information, equipment capabilities, and the nature of our work, the dual receiver mode was implemented. The Radar Timing Generator was enhanced to control transmit and receive polarization by adding two eight-bit shift registers to control the transmit and receive sequences independently. Arbitrary Waveform Generators were used to simulate receiver signals and check the implementation of the algorithms.

REFERENCES

Sachidananda, M., and D. S. Zrnic, 1989: Efficient processing of alternately polarized radar signals. *J. Atmos. Oceanic Technol.*, **6**, 1310-1318

Jameson, A. R., 1991: A comparison of microwave techniques for measuring rainfall. *J. Appl. Meteor.*, **30,** 32-54.

Zahrai, A., and D. S. Zrnic, 1993: The 10-cm-wavelength polarimetric weather radar at NOAA's National Severe Storms Laboratory. *J. Atmos. Oceanic Technol.*, **10**, 649-662.